

MODELLING COST AT RISK

A PRELIMINARY APPROACH FOR BEGINNERS

**Sanga Sangarabalan
and
Per-Olof Jönsson**

**All queries should be addressed to the authors at
sanga@newmalden00.freemove.co.uk
perolofjonsson@yahoo.com**

COST AT RISK MODELLING

This is an introductory article for those who are at the initial stage of developing a Cost at Risk (CaR) model. Since it is intended to encourage the application of this approach we have avoided the heavy quantitative aspects of modelling. This is to keep the discussion simple and readable. For those who are interested in knowing more, some basic references are given at the end of this paper, which can then be followed up with more references. The article is divided in to two sections, the first highlighting the general preliminary approach to Cost at Risk modelling (Sections 1 and 2) and the second (Section 3) on the application to a Middle Income Country (MIC) where the authors have developed a simple model.

1. General Overview

1.1 Objective: The objectives in the IMF and World Bank Guidelines for Public Debt Management –first published in 1999– state that debt managers should seek to minimise the cost of debt at a prudent level of risk in the medium term. Many developing and newly emerging market countries have begun to follow this objective and formulate their debt strategy in terms of financing the gap at minimum cost and prudent level of risk. In applying this objective a country must clearly define the main terms, financing gap, costs, risks and the time frame in which the application is carried out. For many countries this is a new experience and therefore may require assistance in understanding CaR, the methodology to be employed, the scope of such applications and the likely constraints to be faced. In this paper we have provided a list of issues and discussed a preliminary approach in building a CaR model.

1.2 Strategy: A strategy will examine many combinations of costs and risks but finally the country would choose a particular cost and risk combination. The various costs and risks are the result of testing various strategy combinations. In other words financing gap can be filled by various combinations of debt in terms of structure (external, domestic) maturity (short term, medium term), currency combinations (US dollar, Euro, Local currency) and interest rate structure (variable rate, fixed rate). It is worth pointing out that in general, there is no strategy that would generate a scenario which will have a unique solution of minimum cost and minimum risk. Therefore what will result from various strategy combinations is a trade-off between cost and risk. Some countries might prefer -in comparative terms - to have higher cost than lower risk and others might prefer lower cost and higher risk.

1.3 CaR and Debt Sustainability Analysis (DSA): Many of the staff from low income countries (LICs) are familiar today with DSA. In the recent past, the Heavily Indebted Poor Countries (HIPCs) have carried out this type of exercise to determine their entitlement, the magnitude (if eligible) and timing of debt relief. The non-HIPC low income countries and the newly emerging countries do also carry out DSA exercises some times jointly with the IMF and World Bank. In fact, almost all Article IV missions include a DSA which is usually carried out by the IMF with assistance provided by the country. Unfortunately, many of the staff involved in DSA exercises believe or are led to believe that debt strategies can be formulated using a DSA only. We on the contrary, believe that DSA is only the first step in looking at external or fiscal sustainability of debt and to carry out a comprehensive debt strategy more work in terms of various tradeoffs between costs and risks is needed. DSA looks at debt burden (stock levels) and liquidity (flows) indicators. CaR, on the other hand, is confined to flow measures in terms of interest and exchange rate costs. It is an appropriate framework for estimating the various combinations of costs and risks from which a preferred strategy can be selected.

1.4 CaR and Other Frameworks: Developed OECD countries have advocated the application of more advanced perhaps more comprehensive methodologies like Asset Liability Management (ALM)

framework and Value at Risk (VaR). Though these may be more appropriate in developing optimal debt strategies, at the present time many of developing and newly emerging countries are not in a position to adopt these frameworks easily. As and when better data, competent staff and sounder understanding of issues and measures are attained, countries can advance to more sophisticated level of application. We feel that though the main objective of 'minimising cost at a prudent level of risk' remains the same for all countries, each country has to work on this at its own pace. Alternatively, countries should not find excuses to deliberately delay in carrying out the required level of analysis by continuously focusing on their constraints and inability to progress. All countries can embark on this type of work taking a step by step approach, moving gradually from simple applications initially to more sophisticated frameworks later.

2. Debt Structure Costs and Risks

2.1 Debt Structure: In more developed countries, financing of the gap is mainly through domestic sources and therefore debt stock is dominated by domestic debt instruments. External debt is usually a small percentage of total debt, largely as a result of issuing internal bonds that are purchased by non-residents. However when you examine the total debt of LICs and newly emerging market economies (NEMCs) the composition and new borrowing is dominated by external debt. This may change in the future as many of the low income countries having obtained significant amounts of debt relief via HIPC and Multilateral Debt Relief Initiative (MDRI) which will probably reduce the existing external debt. In the future, these countries may borrow from external and domestic sources in more equal proportions. The main point here is that developing and emerging market economies will have a different debt structure to those of the advanced countries by having a combination of external and domestic debt and in several currencies. The types of debt incurred is also different in developing countries as they contract more multilateral and bilateral (including mixed/export credits) debt as opposed to international bonds raised by developed countries. Also, the multilateral debt for LICs tends to be on concessional terms while NEMCs can only obtain either a combination of concessional loans and market based loans or the latter depending on their income level. In summary, LICs would have a significant amount of external debt (likely to continue in the future) in their portfolio of borrowing. These loans will be on concessional terms while their domestic borrowing will bear market interest. Emerging market countries also fill financing gaps by external and domestic sources though the external sources such as loans and bonds will carry market rates of interest. A typical list is given below;

- Multilateral loans (market rate of interest) / credits (concessional interest rate)
- Bilateral loans/export credits
- International bonds (mainly NEMCs)
- Commercial bank loans(mainly NEMCs)
- T-bills of various maturities – 91, 182 and 364 days
- Treasury bonds of various maturities – 2, 3, 5, 10 years

2.2 Costs: There are three important issues in costs that have to be understood when developing CaR models. The first, many are familiar with measuring cost 'when the transaction of payments' takes place. However in defining real cost, the accrual accounting concept should be used. In other words, for example, a 3-year zero coupon bond may not have any cash payments during the three years but when measured on accrual basis a representative cost has to be shown for every year. Second, costs have to be measured in terms of market values (or fair values) rather than historical values. Third, both unrealised gains and losses due to exchange rate movements must be included in the cost. This means that losses and gains on capital amounts (on unrealised basis) had to be taken in to consideration every year.

It may be difficult for staff that are familiar with cash based accounting to move quickly to accrual basis of accounting although the latter is the more appropriate approach. They may need more time to understand, analyse and calculate all costs on accrual and market basis. If more time is needed then

we advise that they carry out CaR on a cash basis initially and later on move to accrual based framework. However, the concern here is that approach based on a cash basis may not lead to optimal strategies. We feel that countries can start off with a cash basis framework but take a cautionary approach to using those results in determining an optimal strategy.

2.3 Risks

Given the structure of debt of LICs and NEMCs, the typical risks faced by these countries are;

2.3.1 Market risk: There are two main variables in this category, namely interest rate risk and exchange rate risks. Interest rate risk is prevalent in instruments (both external and domestic loans and bonds) that have floating interest rates. A country may also face exchange rate risks if a significant portion of repayments are to be made in many currencies that may appreciate against the local currency.

2.3.2 Disbursement risks: On the external side, planned borrowing in terms of loans to fill the gap may differ from the actual disbursement / raising of finance with either shortfalls or delays. This may lead to finding alternative sources externally or domestically at higher costs. For example, a donor postpones a disbursement since all conditionality may not have been met. This creates a difficult situation for the debtor, especially in the case of general budget support, when the expenditures have to be financed from other sources instead.

2.3.3 Credit risks: A government is typically exposed for credit risks when conducting on-lending and issuing guarantees for investment projects. A government could also be exposed to credit risks through the use of derivatives. In general, a country should continuously monitor the creditworthiness of its counterparties.

2.3.4 Re-financing risks: Re-financing risks are defined to be risks faced by a country that is unable to find resources for maturing debt payments or find them only at a high cost.

2.3.5 Operational risks: These are risks due to poor recording, transferring data from one system to another, poorly verified / validated data that is subject to over-repayment risks, operational failures including legal risks etc.

It is possible to measure all these risks and aggregate them in to a single risk measure. However, this is a difficult task that needs the application of a more complex methodology and sophisticated modelling techniques. We recommend for beginners of CaR to confine their forecasts based on market risk (interest rate and exchange rate risks) and refinancing risks.

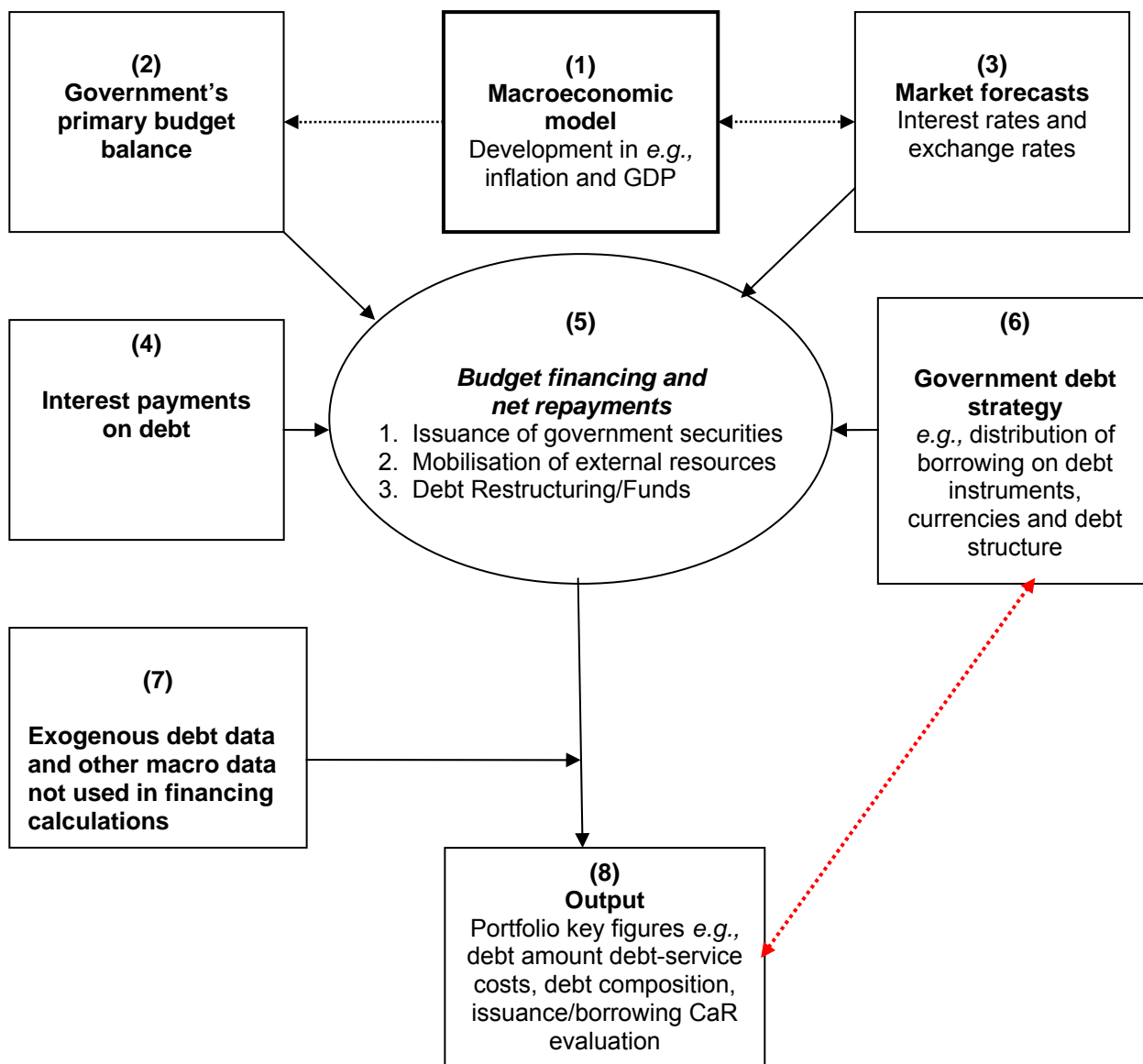
2.4 Modelling Cost at Risk¹

2.4.1 In general, absolute CaR is defined as 'the maximum interest costs with a probability of 95 percent in a year'. Relative CaR is the difference between absolute CaR and expected interest costs in a year. However for developing countries, the costs should include the impact on costs due to exchange rate fluctuations as these countries have and continue to have a significant amount of external debt in foreign currency. Having defined the next stage would be to estimate expected costs and 95 percent level cost resulting from risks.

2.4.2 The modelling process is best explained by a diagram consisting of blocks of activities and information flows.

¹ Annual Debt Management reports of Denmark, Sweden, Netherlands and Canada are useful references.

The diagram below shows the main building blocks which in brief contain existing debt and debt servicing costs, primary budget balance, macroeconomic framework and forecasts and forecasts and forecasts and future development in exchange rate movements and interest rates. The important feature to note in this diagram is the link (shown by the red dotted line) between debt strategy and output targets. If the first strategy assumptions do not give desirable outputs then the strategy can be revised and process reiterated until an acceptable trade-off between cost and risk is achieved. Further, this framework can provide the basis for formulating a debt/borrowing strategy.



Box 1: Macroeconomic model: Forecasting macroeconomic variables is an important task of economists working in government and other private agencies. The major variables such as GDP, inflation, exchange rates and interest rates will be the outputs of a model. Some of these outputs will feed into Box 2 and Box 3.

The responsibility of preparing forecasts rests with economists in Ministry of Finance, Central Bank and other Economic and Statistical agencies.

Box 2: This deals with government finance position. After obtaining the basic macroeconomic forecasts and other relevant information government/public sector financial position had to be estimated. It is better

to prepare these forecasts (one baseline and few worst case positions) using the interrelationships of macro variables thus the need to use macroeconomic models. Models not only provide guide to correlations but also will check for consistency and stability of interrelationships. For example, tax revenues will be based on GDP and Import forecasts as these have an impact on direct and indirect taxes. The model has to check for relationship, correlation and stability. The output from this box is the primary budget balance (excluding interest payment).

The responsibility of preparing forecasts rests with economists and fiscal analysts in the budget department of the Ministry of Finance.

Box 3: The box on market forecasts for interest rates for various instruments and exchange rates for various borrowings is probably the most important numbers as far as debt staff are concerned. In a typical developing country or newly emerging economy, there is likely to be two sources of financing available for funding the fiscal gap i.e., external and domestic.

- On the domestic side, interest rate / yield forecasts are usually required for all the domestic debt instruments such as 91-day T-bill, 182-day T-bill, 364-day T-bill, 2-year Treasury bonds, 5-year Treasury bonds etc. In addition, there are many emerging market countries issuing domestic debt instruments in foreign currencies and floating rates.

- On the external side, countries borrow from International Financial Institutions (IBRD, IDA, ADB, EBRD, EIB etc.), Export Credit Agencies, commercial banks and by raising finance through the issuance of international bonds. The maturities of these instruments can vary from 5 to 40 years and except for concessional loans, market rates (both fixed and floating) apply. Interest rate forecasts are required for the following;

- 1) Short term interest rates
- 2) Interest rates for medium and long term maturities

The interest rate forecasts of various maturities have to be prepared by the debt staff. Central Bank staff might produce short term forecasts but usually never engaged in long term forecasts.

Exchange rate forecasts are also needed since the external loans and international bonds will be usually denominated / paid in major foreign currencies. In general, the three major currencies in which most countries carry out their transactions are US dollar, Euro and Yen. Countries need to forecast the exchange rates between their national currency and the major foreign currencies.

The responsibility of preparing forecasts rests with debt staff for interest rates, and the responsibility of forecasting exchange rates in the short and longer term lies with the Central Bank and Statistical agency.

Box 4 represents interest payments which together with primary balance (Box 1) gives the overall budget position.

The responsibility of preparing forecasts on interest payments lies with the debt staff.

Box 5 represents the budget balance and net repayments of principal (both domestic and external). In addition other funds and operations that are related to debt stocks should also be included in estimating the overall net financing gap to be filled. Net financing also takes in to consideration restructured debt including buybacks, assumption of other debt by government, privatisation funds used for debt stock reduction, debt swaps etc.

The responsibility of preparing forecasts rest with debt staff.

Box 6 denotes strategy formulation; borrowing mix, preference of maturity structure and currency composition. Various strategies will be input to Box 5 and outputs obtained. If outputs are not the desired

ones then a different strategy containing different compositions of external/domestic, foreign/local currency debt, fixed/variable interest rate will be input until a desired output is obtained.

The responsibility of preparing various strategies rest with debt staff.

Box 7 contains relevant economic data that will be used to calculate the ratios and indicators.

The responsibility of providing this information rests with economists in the Ministry of Finance, Central Bank and other Economic and Statistical agencies.

2.5 Forecasting Interest Rates and Exchange Rates²

In this section we have briefly summarised the common models that are used by OECD countries and other methods used in forecasting interest rates and exchange rates. Since interest rate forecasts are more needed in analysing costs and risks in OECD countries, most of the staff in these countries focuses mainly in the development and application of interest rate models. Outputs of such models will be input to determine CaR.

2.5.1 Interest rate forecasting models; OECD countries employ two classes of models namely, linear factor models (affine models) based on spot rates and forward rate models. Both categories of models must satisfy the basic financial / economic consistency requirements. This means that forecasts should not 'explode' but move within realistic levels. A key characteristic of stability is explained by 'mean reversion'. Second is that interest rates are positive with the minimum value of zero (cash equivalent). A typically applied linear model is the Cox-Ingersoll-Ross (CIR) model.

2.5.2 The linear (affine) model can be simply written as;

$$\text{Change in spot rate} = \text{mean reversion component} + \text{stochastic component}$$

Having fitted a good equation for change in spot rates, longer term rates can be estimated by fitting the best correlated equations between longer term rates and spot rates. Stochastic component can be further broken down and explained by more than one factor. For example a two factor model will represent changes in the gradient and shift in the yield curve. Since empirical studies have shown that one factor model does not explain all the movements in a yield curve, additional factors have to be introduced. At maximum, three factors are sufficient to explain the parallel shift (level) in the curve, the changes in the gradient and the changes in the shape of the curve over time.

2.5.3 The second type of model uses forward rates to develop the entire forward yield curve. This is usually carried out by the extraction of zero-coupon and forward interest rates from coupon bearing bonds. However, it is worth noting that the forward rates are time dependent and therefore dynamics have to be introduced to capture the changes in the forward rates.

Which class of models perform better? Evidence is mixed and in some cases the linear model performs better for the short and medium term forecasts while forward rate models perform better for longer term maturities.

2.5.2 Outside of these models one can also look at other time series and econometric forecasting models. On the time series models, Autoregressive-Moving Average (ARMA) models and Vector Auto Regression (VAR) models can be employed to forecast interest rates. In some cases volatilities can be better estimated using General Auto Regressive Conditional Heteroskedasticity (GARCH) model. It might be also useful to introduce other economic variables and combining them with time series models. A more comprehensive approach would be to forecast interest rates through macroeconomic models. One such approach is to link interest rate determination with fiscal balances and other macroeconomic variables.

² References on interest rate and exchange rate models are given at the end of the paper.

2.5.3 Exchange rate forecasts: As mentioned earlier exchange rate forecasting is not the responsibility of the debt analyst and these forecasts should be carried out by other staffs usually employed in the research/statistics departments of the Central Bank. Essentially, forecasts will be only needed for one exchange rate, preferably the most dominant traded foreign currency and its movements against the local currency. In many countries this tends to be the US dollar versus the local currency. Local currency movements against other foreign currencies such as Euro and Japanese yen can be estimated by cross rate forecasts between say US dollar, Euro and Japanese yen. The forecast on cross rates are more readily available via many financial publications.

2.5.4 Like interest rates, exchange rates also have short term exchange rate movements and longer term exchange rate forecasts. For shorter term forecasts, once again we can use spot rates and forward rates. To date, many developing countries have moved away from fixed rates to more flexible exchange rate system under a liberalised environment. The more familiar theories that underpin short to medium term forecast are the Covered Interest Parity (CIP), Uncovered Interest Parity (UIP), Forward Rate Unbiasedness (FRU). The theories are largely based on differential investment in assets denominated in different currencies. Following these theories and linking spot rate to forward rates, short to medium term forecasts can be made.

2.5.5 For longer term rates, the principle of Purchasing Power Parity (PPP) is a more accepted approach which is based on economic fundamentals of external sector developments. In this approach nominal exchange rate movement between two currencies offset differential long term inflation across the two countries. However many investigative studies have shown that PPP hold true over a long term of 10 years or more. As in interest rates here too, time series and econometric techniques can be applied.

2.5.6 General Comments: Those countries that are embarking on CaR modelling will need to forecast market based interest rates for various maturities and exchange rates for various currencies. There are a number of points worth remembering;

- realism and consistency in forecasting
- examine the data well – the series may be short or not homogenous
- models have to represent basic pre-requisites
- ensure correlations are maintained
- start with simple models and move on to sophistication later on as you progress

2.6 Costs and Risks

As mentioned earlier, the two types of cost and risks considered for CaR are market risks and re-financing risk. A debt analyst should estimate future cost as 'expected interest cost' and risk as 'cost at 95 percent confidence level'. Relative cost is the difference between 95 percent costs and the expected cost. Almost all OECD countries use 3 main indicators with some variations and these are; **interest rate fixing** (usually on an annual basis) **average fixed interest period** (duration for the entire horizon under consideration) and **redemption profiling**. As mentioned earlier some variations are evident; for example the Danish model uses McCauley duration while the Dutch model uses modified duration. However, it is important to point out that the principles in the definition and broad output/target measures remain the same.

2.6.1 **Interest rate risk:** The most popular approach for examining interest rate exposure is **the interest rate fixing**. This is a measure signifying a risk against floating interest in the portfolio, a higher the duration the lower the risk. Interest rate is fixed on a yearly basis on the following;

- amount of normal redemptions / amortisations
- floating rate debt
- swaps and
- debt buybacks

Additionally, many countries also set targets for the **average fixed interest period** for the entire portfolio. It does not provide information on dispersion of the interest rate exposure over time. By setting a target with a small interval, one can look at various portfolio strategy combinations and how it affects the target.

2.6.2 Exchange rate risk: Depreciation of the currency can lead to higher payments for amortisation / redemption payments and interest payments in local currency. Exchange rate forecasts have to be made and then be applied to interest payments that are denominated in foreign currencies. Similar application to principal payments will give the amount of refinancing needed in local currency.

2.6.3 Re-financing risks: These are risks faced by debt managers when large amounts of amortisation/redemptions take place in a particular year. This will be based on a priori knowledge on the level of re-financing risk that can be dealt comfortably. In order to avoid large amounts of re-financing debt staff can formulate a desired redemption profile. First, set a benchmark on the maximum amount per year, i.e. a ratio of amortisations / redemptions to total debt outstanding would be a good start. Having decided on this, a smooth redemption profile can be drawn up. This profile has to be continuously updated as forecasts become actual, and when the two differ over time. Quickly re-smoothing changes maturity structures and consequently duration and interest costs. In essence, the various risks associated with Re-financing, interest rates and exchange rates are inter-related either directly or indirectly.

2.6.4 Reporting of CaR: CaR is a measurement of interest costs (including external interest converted to local currency payments), the burden of this in the budget should be represented as a ratio of budget revenue or GDP. Ideally both ratios should be presented for expected costs and for the 95 percent level cost, the latter is the CaR. In addition one can also report on the relative CaR i.e., the difference between absolute CaR and expected cost.

3. An Example of a Simple CaR Model

3.1 General

The relevant risk factors are modelled according to some generally accepted and consistent techniques so that forecasts of expected outcomes and outcomes within a two standard deviation interval are arrived at. Ideally, correlations between the risk factors should be accounted for.

A simple debt model can be developed in Excel in which the risk factors is applied on the actual debt and different scenarios for new borrowing during a 5 year period on a quarterly basis. The model calculates the result in terms of expected cost and risk for each scenario. The expected cost is the result of calculations based on the expected development of the risk factors. It is assumed that there is a simple, linear relationship between the development of the risk factors and the costs in the sense that increased interest rates and depreciation of the currency leads to correspondingly higher costs. Thus, two standard deviation (95 percent confidence level) higher risk factors should lead to two standard deviation higher costs. This assumption greatly simplifies the computations.

Unlike standard CaR models the cost definition in this model is aimed at being on an accrued and not cash basis. Thus, interest cost should be calculated as accrued interest. However, due to practical, computational reasons the interest payments are calculated as a cash flow. If the actual and simulated loans have annual coupons there will not be a significant difference between an accrued and a cash concept. If on the other hand zero-coupons bonds were part of the analysis an accrued concept should be used. Consequently, the exchange rate differentials on the capital amounts are measured on an accrued basis, i.e., when foreign exchange rates change the resulting change in value of the debt is calculated as exchange rate differential.

The rationale for following an accrued concept is the following. Firstly, it is consistent with generally accepted definitions of cost, which is always an accrued concept. Secondly, it becomes possible to analyse the result within a shorter time horizon. Looking at only the cash payments and the

consequences for these under different scenarios for the risk factors requires a very long time horizon in order to take account of long dated foreign currency debt.

3.2 Data Sources and Scope of Model

All information of the actual contracted debt should be easily retrievable from any debt recording system, either through standard reports or through copy and paste of cash flows into Excel. The scope of the model was to cover the central government debt, but it could easily be extended to more comprehensive coverage of debt, i.e., general government or overall public sector. Forecasts, on a quarterly basis, for interest rates (various instruments) and exchange rates (foreign currency debt) were completed.

3.3 Risk Factors

The risk factors, modelled along the lines presented above, are inserted in the debt model on a quarterly basis. Not all actual risk factors need to be explicitly forecasted. In some cases a forecasted risk factor could be used as proxy for the real risk factor, i.e., the forecasted interest rates for Euro linked domestic borrowing could be also used for Eurobonds. Other risk factors were not forecast as the impact was deemed to be insignificant.

3.4 Primary Deficit and Privatisation Proceeds

The model uses as input quarterly forecasts of the primary deficit and privatisation proceeds. In practice yearly forecasts could be used with estimated allocation for the quarters based on past experience.

3.5 Actual Redemptions

Redemptions on a quarterly basis for all the actual debt transactions are inserted into the model. All types of debt instruments are categorised by currency and fixed / floating and are individually shown in the model. All foreign currency debt transactions are shown in original currency and converted into local currency. Quarterly redemptions for fixed rate and floating rate were calculated.

3.6 Actual Interest Payments

The interest payments on the actual debt should also be categorised in to an instrument type, currency and fixed / floating basis. However, only the fixed interest rate payments need to be inserted into the model. For floating rate interest payments, the model calculates the interest payments based on the forecasted interest rate. It is assumed that all floating rate debt in the debt stock is paid quarterly or semi annually, depending on which type of floating rate is most frequent. 3-month US dollar interest rate and 3-month Euribor rate were estimated and input to the model. Thus, the absolute costs for the floating rates will not be exact, but this is seen as acceptable since the purpose of the model is to calculate the difference in costs between different scenarios rather than the absolute costs.

3.7 Simulated Interest Payments

The amount of new borrowing to meet the financing gap and the interest payments for each quarter was estimated. Bonds on a fixed interest are assumed to have annual payments starting one year after the bond was issued and with the corresponding interest rate forecast. Floating rate debt is assumed to be paid on a quarterly basis, so the interest rate is fixed in one quarter and paid in the next quarter.

3.8 Forecasting of Interest Rates and Exchange Rates

Though a country may prefer to have a more sophisticated model that can capture many features of the debt situation and provide realistic forecasts, in reality most emerging market and transition countries face number of constraints that do not allow them to carry out the modelling process at a desired level. The Middle Income Country (MIC) faces a number of constraints that are given below;

1. In terms of debt management it adopts simple rules of financing without any swaps or buybacks.
2. A relatively short series on past data for many of the variables.
3. The economy is in transition and therefore the data is not completely homogeneous.

These constraints were identified in the MIC and relevant techniques were used to prepare quarterly forecast that were input to the simulation model. Forecast covered a period of 3 years from 2007 to 2009 (see Appendix for details). If the calculation needs annual forecast then quarterly forecast can be easily annualised.

3.9 Borrowing Requirement

The total borrowing requirement is calculated as the sum of the primary deficit, privatisation proceeds, redemptions and actual and simulated interest payments and presented on a quarterly basis.

3.10 New Borrowing

New borrowing is simulated on a quarterly basis to meet the borrowing requirement. The model could, for example, allow for floating and fixed rate borrowing for 5 and 10 years in local and foreign currency (Euro). The borrowing from T-bills is, for simplicity reasons, assumed to be replaced with floating local currency borrowing, where the interest rate is based on the 91-day T-bill forecast.

3.11 Actual and Simulated Outstanding Debt

The actual outstanding debt is calculated and presented on a quarterly basis for the 5 year period. The simulated debt is shown on a loan-by-loan basis. All instruments are presented per currency and fixed / floating and all foreign currency debt is shown in both original currency and in local currency.

The total debt is presented in terms of foreign and local currency, both in absolute terms and in relation to the total debt. The floating / fixed composition, the market value, the duration and the interest rate fixing amount (redemptions + floating amount) for 12 months ahead is also estimated.

3.12 Market Value and Duration

Market values and duration are calculated for the main fixed interest rate instruments on a loan-by-loan basis, both actual and simulated. New 5-year and 10-year bonds are assumed to be issued every year. The bonds issued one year are expected to have average market interest rate during the year. The calculations of market value and duration are, for practical and computational reasons, done at the end of each year. For smaller amount of fixed interest rate loans the duration is not calculated on a loan-by-loan basis but approximated to the instrument type. The estimations are based on the maturity and are not affected by the changes in the market interest rate.

3.13 Total Costs

Total cost per quarter is calculated as the sum of the interest payments and the exchange rate differentials in Euro, US dollar, Japanese yen and Swiss franc based on an unrealised basis. For other currencies, no exchange rate differentials are calculated due to the small amounts of outstanding debt.

When comparing different borrowing strategies the costs are aggregated for the whole period.

3.14 Borrowing Strategies

A number of different borrowing strategies could be analysed in the model. The strategies should be chosen in order to provide a few, clearly distinctive outcomes with respect to composition of the debt in terms of domestic and foreign currency borrowing and maturity structure.

3.15 Risk Factor Scenarios

The cost for each borrowing strategy during the 5 year period is analysed under two different set of risk factor scenarios;

- expected development of risk factors
- expected development + two standard deviations

In the MIC, the Central Bank had been quite successful in pegging the local currency to Euro. This monetary regime also put additional pressure on the domestic market, making the local currency interest rates generally higher and more volatile than if a floating exchange rate regime had been chosen. However, the main concern of the Central Bank with a large foreign currency debt is not the small variations around the target level, but what would happen if the Central Bank, for some reason, decided to promote or accept a large depreciation of the local currency. Thus, an additional risk factor scenario, a purely deterministic scenario -with out any probability attached to it- was developed where the local currency is assumed to gradually depreciate 20 percent over the forecast period against the Euro.

3.16 Results

The results of the borrowing strategies under different sets of risk factor development could be summarised in a number of scenario outputs;

- expected cost scenarios
- increased cost for the two risk scenarios (2 standard errors)
- and the deterministic scenario (20 percent depreciation)

Given the general type of yield curve, which is upward sloping short term maturities tend to have lower cost than longer term maturities. The strategy where part of the borrowing is in Euro results in lower cost due to the assumed lower interest rate in Euro than in local currency. In terms of risk, according to the probabilistic approach with two standard deviations, the options with Euro borrowing and with medium to long term maturity seem to be most favourable. This result may appear surprising since foreign exchange borrowing intuitively should be more risky than domestic borrowing. However, due to the historically stable relationship between the local currency and Euro, the risk associated with Euro debt will be relatively low and relatively less important. Our scenarios show overall the more important factor is the historically high volatility in domestic interest rates. Due to the latter effect the domestic options generally give rise to higher risk.

However, a more risky scenario is observed when the Euro appreciates by 20 percent against the local currency. This option is not unrealistic, even though there are no reasons for this to happen as the country is tracking closely the Euro. Nevertheless, it was important to consider this option and include it as one possible debt strategy outcome.

Appendix: Forecasting Exchange Rates and Interest Rates; Quarterly Forecasts from 2007 to 2009

X1. One set of important inputs are forecasts on interest rates and exchange rates on a quarterly basis for the period 2007 to 2009. The choices of forecast are restricted because of the short time series of the past and also the future. For this reason short term forecasting technique of Autoregressive-Moving Average (ARMA) models were chosen. Unlike econometric models which explain future movements in a particular variable by relating to other variables, these models provide forecast of variables that are based on their own past values. Therefore, in these models interest rate movements are explained by movements of this variable in the past and the same principle applies to exchange rates etc. Within time series models many variations are possible; linear models, non-linear models, models with smoothing applications, dynamic/ static simulations, deterministic / stochastic simulations etc. The choice in any one of these is largely related to reliable quantum of data and the homogenous state of the economy.

Simple linear time series models were used to forecast exchange rates and interest rates for the MIC. No correlations effects between different exchange rates or interest rates were taken explicitly in to account in the estimation process. The list of variables is as follows;

X2. Exchange rates: Five main exchange rates were identified and forecasts were carried out using ARMA type models. These are;

- 1) Euro versus Local currency
- 2) US dollar (USD) versus Local currency
- 3) Japanese yen versus Local currency
- 4) Pound sterling versus Local currency
- 5) Swiss franc versus Local currency

When the final equation was chosen quarterly forecasts were made with 2 standard errors (roughly equivalent to the 95 percent confidence level) and were input to the debt simulation model. There are other currencies (such as Swedish kroner, Norwegian kroner, Kuwaiti dinar etc.) used in debt service transactions but the amount is small and therefore simple projections were used to forecast these. Hence, the ARMA application was only adopted for the 5 currencies mentioned earlier.

X.2.1 Forecasting exchange rates: First, a relevant historical time period was chosen. For all currencies except the US dollar, quarterly time series from 1995 first quarter to 2006 second quarter was used. For the US dollar, because of its high appreciation in 1995 to 2000 and depreciation since 2000 until now, a more realistic time frame from 2000 first quarter to 2006 second quarter was chosen. For Euro, Japanese yen, Swiss franc and Pound sterling data, 44 time points were while for the US dollar, only 26 time points were used. In constructing the quarterly data series monthly mid point averages were used. Historical data of the exchange rate was taken from the published archives of the Central Bank; in the case of the Japanese yen, the JPY/Local currency the cross rate was obtained indirectly via the USD/JPY files published by the Central Bank of Japan and USD/Local currency published by the Central Bank.

X.2.2 Chosen equations for exchange rates: Econometric software named Econometric Views (E Views), was used to carry out all the estimation exercises. The method of Ordinary Least Squares (OLS) was used for estimating equations that explained movements in exchange rates. In addition to a constant, Autoregressive (AR) up to 4 quarters and Moving Averages (MA) up to 2 quarters were tested. In selecting an equation for forecasting purposes, Significance tests (adjusted R^2 and 't' ratios) and Unit root tests were examined. Unit roots that are less than 1 provide 'stationarity' conditions for a time series equation. Having selected the most appropriate equation, forecasts for the expected value and values for 2 standard errors representing the 95 percent confidence limits were prepared.

X.3. Interest rates: The following interest rates (based on yields and coupon rates) were forecast;

- 1) 91-day T-bill
- 2) 182-day T-bill
- 3) 364-day T-bill
- 4) 5-year domestic Local currency bond
- 5) 10-year domestic Local currency bond
- 6) 5-year Euro-indexed domestic bond
- 7) 10-year Euro indexed domestic bond
- 8) 5-year Euro bond
- 9) 10-year Euro bond
- 10) 3-month US dollar interest rate
- 11) 3-month Euribor rate

X.3.1 Forecasting T-bills: For 91-day and 182-day T-bills, data on yield is available on a quarterly basis from third quarter of 2000 to second quarter of 2006. Those are the results of the T-bill auctions compiled by the Ministry of Finance. However, the data points are much less (only 24) when compared to exchange rate data. For 364-day T-bills, the data series is further shortened from third quarter 2002 to second quarter 2006, thus giving only 19 points. The point to remember is that small sample size reduces the reliability of estimates. On the other hand increasing sample size does not increase the reliability of estimates if the sample was drawn from two different periods of economic environment. In other words, time series is representative of reality when taken from a homogenous economic environment.

X.3.2 Chosen equations for yields: A fairly similar approach to exchange rates was used in forecasting T-bill yields on a quarterly basis from 2007 to 2009. Similar to exchange rate analysis, ARMA models were tested on T-bills. A small difference being in addition to autoregressive and moving average variables, inflation (retail / consumer price index) was also used as an exogenous variable in the estimation process. Due to this inflation had to be forecast as well and this was done through ARMA models.

X.3.3 Forecasting 5-year and 10-year Euro, Euro-indexed and Local currency bonds: For 5- and 10-year Eurobonds, data was obtained from German Eurobonds series. The series published by the Bundesbank gives average monthly yields for the benchmark bonds with a remaining maturity of respectively 5- and 10-year; the monthly average was converted to quarterly data. The time period covered was from first quarter of 2000 to third quarter of 2006. For domestic bonds for both Euro-indexed and Local currency, a long series is not available. As these instruments are quite new, data is scant and therefore a different method had to be adopted for forecasting. However, the few data available on yields both in the primary market and secondary market was used with judgement. The data of the primary market were obtained from the local Stock Exchange where the prospectus indicated the price of issuance. The data of the secondary market was tracked via the publications from Reuters. The exception to this was the availability of data for 5-year Local currency bonds; yields were calculated and published by the Central Bank. The data was available every two months starting from May 2003.

X.3.4 Chosen forecasted equations for yields: Once again ARMA models were used for Euro bonds for both 5 and 10 years. Expected outputs and 2 standard error outputs were estimated. For Euro-indexed domestic bonds a correlation was established between Eurobonds and Euro-indexed bonds. For 5 year Euro-indexed bonds, the margin was 100 basis points above 5-year Eurobonds. For 10-year Euro-indexed bonds, a margin of 130 basis points was added to the 10-year Eurobond. 5-year Local currency bonds were forecasted via ARMA models while 10-year Local currency bonds are based on a margin of 50 basis points above a 5-year Local currency bond.

X.3.5 Forecasting short term rates for Euro and US dollar: Data series over a long period is available for three month rates on Euro Inter-Bank Offer Rate (Euribor) and Eurodollar deposit rates which time series

are published respectively by the Bundesbank and the Federal Reserve. However, for consistency and reliability, rates starting from year 2000 were used.

X.3.6 Chosen forecasted equations for interest rates: ARMA models were tested on these rates an appropriate equation was selected for forecasting the expected value and 2 standard errors, on a quarterly basis from 2007 to 2009.

References

Term structure of interest rates;

- 1) Cox, J. C., Ingersoll, J. E., and Ross S. A., (1985), A Theory of the Term Structure of Interest Rates, *Econometrica*, vol.53, no.2, pages 385-407
- 2) Heath, D., Jarrow, R. and Morten, A., (1992), Bond Pricing and the Term Structure of Interest Rates, *Econometrica* 60:1, pages 77-105
- 3) Bolder, David J., (2006), Modelling Term Structure Dynamics for Risk Management; A Practitioner's Perspective, Bank of Canada Working Paper 2006-48, December 2006
- 4) Towards a More Complete Debt Strategy Simulation Framework, Bank of Canada Working Paper 2002-13, May 2002

Exchange rate forecasting models;

- 5) For Covered Interest Parity, Taylor, M. P., (1987), Covered Interest Parity: A High Frequency, A High Quality Data Survey, *Economica*
- 6) For Uncovered Interest Parity and Forward Rate Unbiasedness, Frankel, J. A., (1980), Tests of Rational Expectations in the Forward Exchange Market, *Southern Economic Review*, pages 1083-1101
- 7) For Purchasing Power Parity, Ardeni, P. G., and Lubian, D., (1991), Is There Trend Reversion in Purchasing Power Parity, *European Economic Review*, pages 1035-1055

For Cost at Risk models, see Annual Debt Management Reports and Relevant Research Publications from websites at;

Denmark: www.nationalbankenkenken.dk

Canada: www.bankofcanada.ca

Netherlands: www.dutchstate.nl

Sweden: www.rgk.se

United Kingdom: www.dmo.gov.uk